Safety @ Risk?

Giannis Giannakopoulos[1], Dimitris Bouros[2], Nikolaos P. Ventikos[3]

National Technical University of Athens,
School of Naval Architecture and Marine Engineering,
Laboratory for Maritime Transport, Greece
1)nm00024@mail.ntua.gr, 2)nm00604@mail.ntua.gr, 3) niven@deslab.ntua.gr

Abstract

The following paper focuses on societal risk criteria and more specifically on the general principles as well as the mathematics that underlie the criteria that are in use today. We discuss the main concepts involved, such as risk aversion, the perception of risk, the importance of the risky activity to economy and society. The main weaknesses and virtues of existing criteria are stressed and proposals for future actions are made.

Keywords:
risk acceptance criteria, F-N curves, disutility functions, perceived risk, risk-benefit analysis

1. Introduction

The birth of nuclear and chemical industry in the 1960s brought a new need for safety procedures and standards that would ensure that the development of such industries would not transform from a technological miracle to a hell-like nightmare. It was about then, that risk left the world of gambling and finance to reside in the world of engineering. It was clear that if one wanted to have a picture of the drawbacks of the new technologies, they should be able to identify the hazards connected to these technologies, calculate their probabilities and then the probabilities that those hazards produce certain consequences.

Then a simple formula could describe the 'risk' of this technology, as in,

$$\boldsymbol{Risk} = \sum_{i}^{n} \boldsymbol{p}_i \times \boldsymbol{C}_i \quad (1)$$

where p_i is the probability of the adverse event i;
C_i the consequences from adverse event i.

Although the above formula might seem simple, the calculations of p_i and C_i can prove to be a very weary job. When there is no statistical data, a thorough modeling of both the probability of the adverse event and its consequences is needed. Even if there is statistical data one should be very careful in its translation to either probabilities or frequencies.

However once one has calculated the risk of an activity what is to do with it? Is it high or is it low? Is it acceptable or not? Such questions made a call for benchmarks, for reference points that would offer a comparison to the risk at hand and if anything else would serve as indicators of whether our risk can be accepted or should be reduced.

As risk analysis established its role in decision-making, more complex questions started to arise. Can a low risk for the individual constitute a high risk for a group of people or for that matter for society as a whole? Can there be one universal criterion that can be applied to all activities or is it that each activity must have its own criteria?

Should the individual's or society's perception of risk be taken into account when evaluating risk or setting criteria?

It was clear that a new chapter was beginning; that of risk analysis and quantitative assessment.

The benchmarks used early on to assess the riks in nuclear and chemical industry may have served their purpose, however today it is more and more evident that such benchmarks paint far from an accurate picture of how the public evaluates risk and what levels of risk it is ready to accept.

The shipping industry only recently started to consider risk based regulations and seems so far hesitant to adopt firm *risk acceptance criteria*. Instead IMO refers to the HSE (the British Health and Safety Executive) benchmarks as indicative but not binding, leaving the calculation of the criteria to FSA [7].

The structure of the paper is as follows:

Section 2 serves as an introduction to the main concepts behind *societal risk criteria*.

In Section 3 we will describe the current practice in the construction of criteria on *F-N* curves and discuss the advantages and disadvantages of the various methods used.

In Section 4 we address the subjects of risk aversion and disaster aversion and how those concepts are frequently confused.

Section 5 discusses the concept of risk Perception and the relation between perceived and effective risk.

Section 6 introduces *risk- Benefit analysis* as a method for setting future *risk acceptance criteria*.

Finally, Section 7 contains our conclusions and insights from the aforementioned tasks.

2. Societal risk acceptance criteria

Societal risk criteria were introduced as a measure of how much risk society, rather than the individual can tolerate, as a result of a specific activity.

For example, an individual risk of 10^{-4} for a population of 10 will result in one death in the next thousand years. The same risk for a population of one million will result in 100 fatalities during the next year. Is society willing to accept such a death toll? What if those 100 fatalities were caused due to one single event? Would that make any difference in society's decision? This is the kind of questions that societal criteria try to answer.

Societal risk criteria are closely associated with F-N curves. Such curves plot the cumulative frequency of N or more fatalities against the number of fatalities, N, in a log-log diagram. The criterion lines are applied as straight lines on the log-log plot. There is an upper and a lower bound.

Between the two, the ALARP (As Low As Reasonably Practicable) principle applies. Risks above the upper bound are considered intolerable and should be reduced at all costs. Risks below the lower bound are considered very small for any risk Reduction Measures to be applied. Inside the ALARP area risks are reduced to the extend that it is practically possible. What is practically possible is currently determined through cost-benefit analysis.

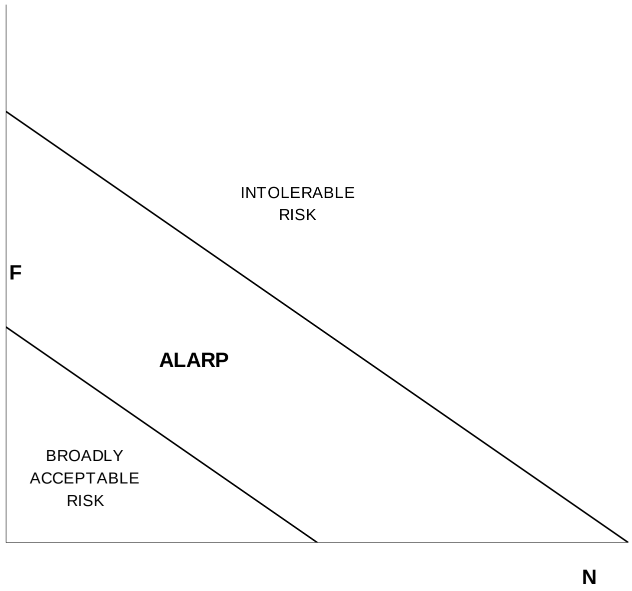

Fig 1. The ALARP area.

We have good reason for not assigning numerical values on the x and y axis. To this day, there has been little agreement on the exact equations of the limit lines even though it is common practice to assume that the lower bound lays two orders of magnitude under the upper bound. Due to this, in the following we will study the construction of the upper bound assuming that the lower largely depends on the shape of the upper bound.

3. Construction of societal risk criteria on F-N curves

Literature offers basically two ways of drawing the upper bound line. Both consist of determining one point and then drawing the line that passes from this point with a slope that has been pre-decided. In both cases the limit line is of the form,

$$F(N) = \frac{c}{N^a} \quad (2)$$

where $F(N)$ is the **upper bound frequency** of accidents with N more fatalities
and c= constant

It is easy to see that c represents the point where the limit line intersects with the vertical axis (since N takes values in $[1, N_{\max}]$), and thus we have,

$$c = F_1 \longrightarrow F(N) = \frac{F_1}{N^a} \quad (3)$$

where F_1 is the **upper bound frequency** of accidents involving one or more fatalities.

Note that the above formula represents a straight line only in the *F-N* diagrams' context, where the axes are in logarithmic scale. If we were to assume linear scale the above equation would translate into a hyperbole. It is obvious that (for a given slope, *a*) the problem comes down to determining a point on the limit-line. Each of the methods takes a different course.

Anchor points

The first one makes use of the so called 'anchor-points'. Those are points on the *F-N* curves that represent experts' judgments on what risks can be tolerated by society and often reflect the amount of risk that has been tolerated in various circumstances. There have been used quite a few anchor points over the years, such as the Canvey point and the ACMH point.

One of the most recent and important *anchor points* is the R2P2 *criterion* that states that '*accidents resulting in 50 or more fatalities should happen no more often than once in 5000 years*' [1], [2]. On an *F-N* diagram this translates to the point (50, 1/5000).

It has to be made clear there is no mathematical formula that produces the aforementioned *anchor points*. They are derived as a result of expert judgement and often as answers to specific unfortunate events such as the Flixborough accident (ACMH point) [1], [2]. These observations raise two important questions:

1. How much can we trust expert judgment? Especially since it is clear that through the years the various anchor points have taken quite different values.
2. If those anchor points where decided upon as answers to specific situations (i.e. accidents of a particular activity), is it legit to use them for the evaluation of other activities?

Calculation of F_1

The second method relies on mathematical formulation in order to calculate the upper bound of acceptance for fatal accidents for a specific activity, hence the definition of F_1.

Vrijling [8] proposed an upper bound F_1 as,

$$F_1 = \left[\frac{\beta_i \cdot 100}{k \cdot N_A} \right]^2 \quad (4)$$

where,

$\beta_i \in (0{,}01, 100)$ is an indicator of the degree to which the individual takes part in an activity voluntarily. Activities with a big degree of voluntariness such as mountaineering suggest large β_is. Hence activities with little or no voluntariness suggest respectively little β_is;
k is an index of the law- makers risk aversion (the proposed value is 3);
N_A is the number of independent installations (for the shipping industry this would mean the total number of ships).

Skjong [12] suggests a different model in order to obtain the upper bound frequency of fatal accidents, $\boldsymbol{F_1}$. He first calculates the maximum number of deaths that can be accepted as a result of an activity (in this case it is the shipping industry).

That is,

$\boldsymbol{PLL_A = q \cdot EV}$ (5) for the persons working on the ship;
$\boldsymbol{PLL_A = r \cdot EV}$ (6) for the passengers of the ship.

Where *EV* is the *Economic Value* of the activity

$$\boldsymbol{q} = \frac{\text{total number of deaths due to work related accidents}}{\text{GDP}}$$

$$\boldsymbol{r} = \frac{\text{total number of deaths due to the activity}}{\text{contribution of the activity to GDP}}$$

This way the economic importance of the activity is taken into account for the calculation of the maximum number of deaths than can be tolerated due to it.

Then $\boldsymbol{F_1}$ can be obtained as,

$$\boldsymbol{F_1} = \frac{\boldsymbol{PLL_A}}{\int_1^{N_{max}} \frac{1}{\boldsymbol{N^a}} \cdot \boldsymbol{dN}}$$

where
a is the slope we chose for the limit line (Skjong chooses a= 1);
N_{max} is the maximum number of casualties that could occur in one single accident.

It should be noted that both Vrijling and Skjong calculate the upper bound $\boldsymbol{F_1}$ taking into account the specific characteristics of each particular activity. Vrijling focuses on the degree of voluntariness in the individual's participation in the activity, whereas Skjong focuses on the importance of the activity for the economy.

The authors of this paper are clearly in favor of the calculation of $\boldsymbol{F_1}$ as opposed to the use of anchor points. Two facts about the calculation of $\boldsymbol{F_1}$ we find greatly appealing:

1. The process of the calculations is clear and ready to be subjected to scrutiny. Anchor points on the other hand offer no chances for re-evaluation, inspection or even criticism;
2. The process is much more activity oriented. That means that the characteristics of the activity are taken into account and thus the limit line is not some generic boundary but a boundary designed for the particular activity.

Determining the slope

After one point of the limit line has been established, the next step would be to decide about the slope of the line, in other words to decide on the value of a. Since a is found in the denominator its use is to reduce the **upper bound frequency** of accidents of N or more fatalities as N gets larger.

In practice a either takes the value 1 or 2. That is, the most commonly used criterion lines are of the form,

$$\boldsymbol{F(N) = \frac{F_1}{N}} \quad (7)$$

or

$$\boldsymbol{F(N) = \frac{F_1}{N^2}} \quad (8)$$

In the literature it is widely considered that a slope of -1 (*a*=1) is risk neutral whereas a slope of -2 (*a*= 2) is risk averse [2]. There are however authors (e.g. Skjong) that claim the slope of -1 to be risk averse as well [12]. IMO proposes a criterion line with *a*= 1 considering it to be risk averse [7]. The terms risk neutral and risk averse reflect the law maker's attitude concerning the magnitude of an accident.

We find that the terms disaster neutral and disaster averse would be more appropriate [14]. A disaster neutral policy considers an accident of 100 casualties as significant as 100 accidents of 1 casualty each. A disaster averse policy considers the first case as much more severe than the second one, and therefore is willing to take harsher measures to prevent great scale accidents even if the total number of deaths in all accidents stays the same.

The authors of this paper are of the opinion that both -1 and -2 slope criterion lines are disaster averse. The proof for that lies in the behaviour of the *f-N* (absolute frequency) criterion lines that can be derived from the *F-N* criterion lines (equation (3)). The complete mathematical calculations can be found in Annex A, where it is proved that criteria with a slope of -1 are despite common belief, disaster averse.

An important observation about criteria with a slope of -1, is that such criteria fail to limit the maximum tolerated number of deaths per year (***PLL***) for an activity.

For the upper bound ***PLL*** of an activity we have,

$$PLL = F_1 \cdot \int_1^{N_{max}} \frac{1}{N^a} \cdot dN \quad (9)$$

Now suppose we choose a criterion line with a slope of -1, that is *a*= 1,

$$PLL = F_1 \cdot \int_1^{N_{max}} \frac{1}{N} \cdot dN$$

$$\Rightarrow PLL = F_1 \cdot \left(\ln N_{max} - \ln 1 \right) = F_1 \cdot \ln N_{max}$$

However we observe that while $N_{max} \to \infty$, $\ln N_{max} \to \infty$ and therefore $PLL \to \infty$. That means that the accepted number of casualties per year will increase infinitely as the potential number of casualties in one accident increases.

We find this to be a serious shortcoming of the -1 slope, as the fact that an infinite number of fatalities could be possible, does not mean that society is willing to accept it. This, in essence, would defeat the purpose of setting criteria altogether.

On the contrary if we are to suppose a criterion line with a slope -2 (*a*= 2),

$$PLL = F_1 \cdot \int_1^{N_{max}} \frac{1}{N^2} \cdot dN$$

$$PLL = F_1 \cdot \left(-\frac{1}{N_{max}} - (-1) \cdot \frac{1}{1} \right) = F_1 \cdot \left(1 - \frac{1}{N_{max}} \right)$$

As $N_{max} \to \infty$, $\frac{1}{N_{max}} \to 0$ and subsequently $PLL \to F_1$. That means that the number of deaths that can be tolerated due to an activity in one year is limited and equals numerically to the upper bound frequency for fatal accidents that come as a result of the activity.

It should be stressed that in all the above calculations we treat the number of deaths N, as a continuous positive variable. This might seem unrealistic, as the result of an accident can only be an integer. The reasons for assigning real positive values to N, are mainly two:

First, when a probabilistic analysis on possible outcomes is performed, the results can well be non-integers [2]. Second, and more importantly, when discussing F-N based criteria it should be kept in mind that in all cases, they are presented as continuous straight lines. This suggests (even if not explicitly) a continuous distribution for N.

Had we wanted a discrete distribution for N, we should also have changed the form of the criterion lines, to what would probably be a set of discrete criterion points.

Finally we have to point out that our choice between a discrete and a continuous distribution for N, will have consequences on the results of our calculations. For example if we need to calculate the upper bound PLL for an activity the sum,

$$PLL = F_1 \cdot \sum_{1}^{N_{max}} \frac{1}{N^{a}}$$

does not equal the integral,

$$PLL = F_1 \cdot \int_{1}^{N_{max}} \frac{1}{N^{a}} \cdot dN .$$

4. Uncertainty aversion as opposed to disaster (magnitude of consequences) aversion

We have seen so far that *F-N* based criteria take into account the magnitude of accidents and are in fact adverse towards accidents of high consequences. However, risk by its definition is not all about consequences. It has to do with the probability (or frequency) of the adverse event as much as it has to do with its outcome. Of course those probabilities (or frequencies) are never certainties. On the contrary there is a certain (smaller or greater) degree of uncertainty associated with each outcome.

Let's suppose two activities A and B that both gave 5 accidents in the past year. All accidents in activity A resulted in 3 fatalities each, giving a total of 15 fatalities.

Activity B gave 3 accidents with one casualty, one with 5 and another one with 7.

The degree of uncertainty about the severity of the outcome is greater for activity B and this is reflected by a standard deviation of 2,53 in activity B as opposed to a zero standard deviation for activity A.

It seems that even though in both situations the total number of casualties is the same activity B is less predictable.

We believe that when setting criteria it might be reasonable to take this unpredictability into account. The question is whether *F-N* criteria can deal with it. It seems that said criteria do not take into account the degree of uncertainty in the accident behavior of activities. One might argue that the matter is well taken care of, with the use of the factor '*a*'. However the slope of the criterion line only shows our aversion towards accidents of high magnitude, not our aversion against the uncertainty of the situation.

Bedford [14], proposes the use of a disutility function in the form of,

$$u = f_N^{\beta} \times N^{a} \quad (10)$$

where
β serves as measure of our aversion towards the uncertainty of the outcome
and a as a measure of our aversion towards the severity of the outcome

It can be easily seen that the above equation with $a = \beta = 1$ and summed over all *N*, gives the PLL of the activity since,

$$PLL = \sum_{N} f_N \times N$$

This disutility function is not a criterion. It is more of a measure of the activity's risk. In fact it can be said that the risk itself (as the product of probability times consequences, $p_i \times c_i$), is a special case of the general disutility function family, $u = f_N^{\beta} \times N^{a}$.

However we feel that there is great potential in its form and in the fact that it allows for modeling an adverse attitude against the uncertainty as well as the severity of an outcome.

5. Perception of risk

Until now, we have discussed about *societal risk criteria* in the form of *F-N* criterion lines and how these criteria try to include in their modeling various aspects such as the economic importance of an activity, the degree of voluntariness, and the severity of the consequences in case of an accident.

We have also discussed the need to take into account the degree of uncertainty about the possible outcomes when evaluating the risk of an activity.

Hence, all above issues are nothing but ways to quantify a notion, already predominant since the 80's, that risk cannot be expressed simply in terms of probabilities and frequencies. Slovic, Fischoff and Starr [3], [4], [11] were among the scientists that suggested that risk cannot be considered outside our perception of it. Voluntariness, familiarity with the activity, the degree of control over the risk, the expected benefit, knowledge of the hazards, the dread for large scale accidents, all play an important part to the person's perception of risk.

However about three decades later it seems that perceived risk pretty much remains a notion to be quantified sometime in the future. Most risk criteria and risk evaluation techniques might try to take into account certain aspects of risk, but never dare to develop a generic model that will be able to take into account all those factors that affect the individual's and society's perception of risk.

Plattner et al [13], propose an interesting formula that can be viewed as a first step towards the right direction. They claim that the perception of risk for an activity, should be a function in the form of,

$$\boldsymbol{R_{per} = f\left(R_{eff}, bias\right)} \quad (11)$$

where,

R_{eff} is the effective risk that can be calculated as,

$$\boldsymbol{Risk = \sum_{i}^{n} p_i \times C_i}$$

bias is our perception about the particular risk;
R_{per} is the perceived risk if we take into account both its effective value and the public's bias against it.

They then suggest that the bias against the risk should be a multiplication factor that has the ability to either increase, reduce or leave the effective risk as is. The formula for R_{per} would then look something like that,

$$R_{per} = R_{eff} \cdot \frac{\sum_{i=1}^{n} \left(paf_i \cdot a_i\right)}{\sum_{i=1}^{n} a_i} \quad (12)$$

where,
paf_i are the perception affecting factors (i.e. voluntariness, control etc);
a_i the relative importance of factor i (for example what does the individual consider more important, that he has little control over the risk or that he expects a high pay off?)

Plattner et al [13] have gone so far as to assign values to the various paf_i's and a_i's by preparing questionnaires and performing public inquiries. We could not say whether those values reflect reality or not. However we feel that if one wants to have a full picture of risk he/she cannot overlook the fact that people seem to view the same effective risks in different ways.

One further question is, 'should risk perception be accounted for in regulations?' The answer is not an easy one to come up with and the relevant discussion in Ball and Floyd [2] serves in our opinion as a good introduction to the issue. We feel that it would be rushed and certainly outside the targets of this paper to propose the use of risk perception factors in regulation. On the other hand it is clear to us that risk and our perception of it are tightly bound together and therefore risk percetion cannot be disregarded without good reason.

6. Risk-Benefit analysis

What equation (12) does not take into account though, is the benefit a person expects by exposing himself/herself to a certain amount of risk. Indeed, benefit plays a crucial role in risk acceptance and risk evaluation.

No risk would ever be accepted if there was not a certain amount of gain involved. This seems to be a common belief for a wide range of scientists from Fischoff [4], to those who support the controversial homeostasis theory [5].

But how is one to adequately model benefit and how could this be implemented in a risk analysis?

Skjong's formula could be viewed as a first step; it does recognize the need to take benefit into account. But does it paint the whole picture? Is it the optimum way to quantify benefit and integrate it in risk analysis?

Fischoff [4] expresses the opinion that anyone directly exposed to risk should also receive a direct benefit for this exposure. This he argues could be anything from tax-cuts, to the reduction of other risks as counterbalance for the risk the person is exposed to, to the sense of ethical satisfaction one gets when helping the community. Whatever this benefit is, it must be made sure that there is some benefit.

It is interesting to see how the medical society addresses the problem of benefit in medical research on human beings. The benefit produced by the research, is divided in benefit for the people who are subjected to the research (and therefore the risk) themselves, and the benefit that goes to third parties such as other patients, future patients, relatives of the research subjects etc [9].

It is made clear that the gain for those immediately exposed to the risk of the research, should constitute the largest portion in the calculation of the total benefit, outweighing the benefit of the third parties.

An interesting example of direct benefit in compensation for an increased level of risk imposed to a group of people, can be found in the Greek policy on the landfill of Nea Liosia.

6% of the municipal taxes of all the habitants of Athens are directed to the Municipality of Nea Liosia. This funding is meant to be used as a direct benefit for all the residents of Nea Liosia and as a compensation for the increased risks due to the landfill.

This does not mean that the money is split directly between the residents, but rather that it is used for the development of the area, thus improving the quality of life of the residents.

In the light of these it seems that the formula proposed by Skjong fails to make that distinction between the individual and the societal benefit. It might be true that the EV of an activity reflects its importance for society, but (when talking about ships) it is not society that risks drowning in case of an accident. It is the ship's crew and passengers. And theirs is the benefit that should be calculated first, at least from safety point of view.

7. Conclusions

In conclusion we feel that today's risk criteria seem over-simplistic. They do not seem to be able to model the multiple aspects that form our perception of risk and most importantly they seem to be single sided as in general (with the probable exception of the criterion proposed by Skjong) they overlook the benefit part of the equation. The current practice is to compare the risks between activities to decide upon their accepted. However this does not say anything about the absolute level of risk that society is willing to accept for a particular activity. For example setting the acceptable levels of risks in shipping, in accordance to road safety would most likely mean unnecessarily high risks for crews and passengers.

We believe that future criteria can only be considered as part of a risk benefit analysis that weighs the gains (first towards those exposed to the risk and then towards society) and losses (in terms of risk) of a particular activity. Such analysis is not to be confused with the current practice of cost benefit analysis for the application of RCOs.

This is the reason we chose to call it risk benefit analysis here, even though it could be viewed as a form of an extended cost benefit approach, or in an even broader context an application of decision theory. Its purpose is not to decide about risk reduction measures but rather to decide about the acceptance of the activity altogether. In this context it could be said that risk benefit lays one step before of what is currently labeled as cost benefit analysis in safety field.

Of course such an analysis might turn out to be a quite tedious task. With Slovic's words [11], '*decision analysis and its variants have a number of potentially serious limitations, perhaps the most important of which is the unavailability of the data needed to complete the analysis*'.

However there are certain aspects of this kind of approach that we find greatly appealing and especially important when setting safety standards. Again as Slovic [11] puts it, '*An important advantage of these methods for decision making in the public sphere is that they are easily scrutinized. Each quantitative input or qualitative assumption is available for all to see and evaluate, as are the explicit computational rules that combine them*'.

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ANNEX A

We will show that criteria with a slope of -1 are disaster averse.

In order to complete the task we will explore the behaviour of the criteria in two cases:

A) towards the risk of having accidents with N or more fatalities

B) towards the risk of having accidents with exactly N fatalities

A) The upper bound risk of accidents with exactly N fatalities is,

$R_N = f_N \times N$ (13)

Where f_N is the upper bound probability of accidents with exactly N fatalities, derived by the upper bound cumulative distribution function, F_N

If we seek to calculate the upper bound risk of N or more fatalities, then this should be the sum, $R_N + R_{N+1} + R_{N+2}\ldots$, or since we assume a continuous distribution for N, it is given by integration of equation (13) from N to N_{max}

Thus we have,

$$R_{>N} = \int_{N}^{N_{max}} f_N \cdot N dN$$

It can be shown [7] however that,

$$\int_{N}^{N_{max}} f_N \cdot N dN = \int_{N}^{N_{max}} F_N dN$$

And so we have,

$$R_{>N} = \int_{N}^{N_{max}} F_N dN ,$$

And since F_N is the upper bound criterion cumulative distribution,

$$R_{>N} = \int_{N}^{N_{max}} \frac{c}{N} = c \cdot ln\ N \Big|_{N}^{N_{max}}$$

$$R_{>N} = c \cdot ln\ N_{max} - c \cdot ln\ N$$

This is the maximum risk of N or more fatalities that can be tolerated.

We go on now to calculate the risk of accidents with $N+1$ or more fatalities.

If we perform the previous calculations only for $N+1$ fatalities, we have

$$R_{>N+1} = c \cdot ln\ N_{max} - c \cdot ln\left(N+1\right)$$

which is the maximum tolerated risk for accidents of N+ 1 or more fatalities.

Let us now compare $R_{>N}$ with $R_{>N+1}$.

It will be proven that,

$R_{>N} > R_{>N+1}$ (14)

For if,

$$R_{>N} > R_{>N+1}$$

We get,

$$c \cdot \left(ln\ N_{max} - ln\ N\right) > c \cdot \left(ln\ N_{max} - ln\left(N+1\right)\right)$$

and since c is positive,

$$ln\,N < ln\left(N+1\right)$$

$$N < N+1$$

the last statement is true and therefore so is (14).

That means that the maximum tolerated risk of accidents with N or more fatalities is larger than the maximum tolerated risk of accidents with $N+1$ or more fatalities.

In fact it can be proven that this applies for criteria in the form of $F(N) = \frac{c}{N^a}$ for any α.

B) Now we will examine the case where we are interested in the behavior of our criterion towards the risk of accidents with exactly N fatalities.

Assume that our criterion line has a slope of -1.

Since we have that, $F(N) = \frac{c}{N}$ is the upper bound cumulative distribution function for N or more fatalities, then the upper bound cumulative distribution function for N or less fatalities will be by definition,

$$\boldsymbol{G(N)} = 1 - \boldsymbol{F(N)} = 1 - \frac{\boldsymbol{c}}{\boldsymbol{N}}$$

and the p.d.f.,

$$\boldsymbol{f_N} = (\boldsymbol{G(N)})' = (1 - \boldsymbol{F(N)})' = \left(1 - \frac{\boldsymbol{c}}{\boldsymbol{N}}\right)' = \frac{\boldsymbol{c}}{\boldsymbol{N}^2}$$

So the upper bound risk for exactly N fatalities becomes,

$$\boldsymbol{R_N} = \boldsymbol{f_N} \cdot \boldsymbol{N} = \frac{\boldsymbol{c}}{\boldsymbol{N}^2} \cdot \boldsymbol{N} = \frac{\boldsymbol{c}}{\boldsymbol{N}} \quad (15)$$

If we perform the same set of calculations only this time for *N+1* fatalities, we have that the upper bound risk of an accident with exactly *N+1* fatalities is,

$$\boldsymbol{R_{N+1}} = \boldsymbol{f_{N+1}} \cdot (\boldsymbol{N}+1) = \frac{\boldsymbol{c}}{(\boldsymbol{N}+1)^2} \cdot (\boldsymbol{N}+1)$$

$$\boldsymbol{R_{N+1}} = \frac{\boldsymbol{c}}{\boldsymbol{N}+1} \quad (16)$$

If we compare (15) and (16) it can be easily seen that,

$R_N > R_{N+1}$

That means that, for criteria with a slope of -1, the upper bound risk of accidents with exactly *N* fatalities is bigger than the upper bound risk of accidents with exactly *N+1* fatalities.

In conclusion, both the cumulative (case A) and the absolute (case B) risk that can be accepted as results of criteria with a= 1, are decreasing as the number of fatalities increases; providing this way the necessary overview for the justification of our approach . To that effect, we consider such criteria to be indeed averse towards increased numbers/magnitude of fatalities.