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Probabilistic Ship Structural Analysis: An Incorrect Way to Compute the Probability of Failure

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There has been a recent paper in the probabilistic ship structural analysis literature that describes an incorrect way of calculating the probability of failure. This short technical note points out this fallacy and suggests the correct way to calculate this probability.

A COMMON approach in the probabilistic analysis of ship strength is the so-called "Level 3," or "fully statistical" approach (see Chang [1] for definitions and more details). The approach assumes that both "ship load" x and "ship strength" y are independent random variables, whose probability density functions (pdf's), $f_d(x)$ and $f_c(y)$, respectively, are known. By "ship load" (also denoted as "demand" in [1]) one traditionally refers to a variable such as the bending moment amidships, or the torsional moment, or generally to any other variable describing a particular way of imposing load on the ship. "Ship strength" (also denoted as "capacity" in [1]) is measured in the same units as "ship load," and typically refers to an upper bound on the load imposed on the ship beyond which "ship failure" occurs.

It is important to clarify right at the outset the scope of this technical note. Two considerations are important in that respect.

1. It is definitely not our aim to fully discuss the merits of the "Level 3" model for the assessment of ship structural reliability—the literature on this and related issues is already very rich (see [1] for a bibliography of over 100 references in this area). Rather, we want to point out an error (rather serious, in our opinion) in the method of calculating the probability of failure (PF) according to this model. We recently spotted this error in [1], and that motivated this note.

2. In all fairness to the author of [1], we cannot rule out the possibility that the error in question first appeared in other papers, probably published prior to [1]. If this is so, fairness dictates that the error should rather be attributed to these other papers. However, given the limited scope of this note, we felt it would serve no constructive purpose for us to search for these other articles, should they in fact exist. Given that [1] is very recent, and is also a state-of-the-art review of ship structural reliability, we decided to base this note on [1] alone.

With these clarifications in mind, we now proceed to our analysis.

According to [1] (see specifically pages 300–301), the method for computing PF essentially consists of the following steps.

Step 1: Superimpose the two pdf's on the same coordinate system (see our Fig. 1, and Fig. 1(*a*) of [1]).

Step 2: Compute PF as the area of overlap of the two pdf's (shaded area of Fig. 1, and Fig. 1(*b*) of [1]).

Step 3: Alternatively, compute PF as equal to

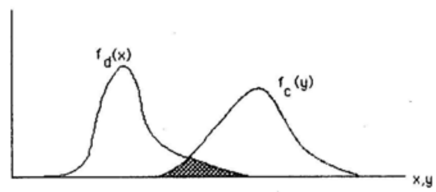

Fig. 1

$$\int_0^\infty f_d(x)f_c(x)dx \text{ (equation (1) of [1])}$$

Reference [1] actually goes on to examine some special cases to the above equation, specifically the cases in which either f_c alone, or both f_d and f_c, are Dirac (delta) functions. In the former case, it is assumed that $f_c(x) = \delta(x - x_2)$ (equation (2) and Fig. 1(c) of [1]), in which case PF $= f_d(x_2)$. In the latter case, it is also assumed that $f_d(x) = \delta(x - x_1)$ (equation (3) and Fig. 1(*d*) of [1]), in which case it is claimed that PF = 0.

Such an approach is incorrect, for the following reasons:

(*a*) First, given that x and y are different variables, superimposing their pdf's on the same coordinate system (Step 1) is meaningless, for the variable on the horizontal axis is ambiguous. The correct way would be to form a joint pdf of x and y on the (x,y) plane (see later).

(*b*) An immediate consequence of the above is that the area of overlap between the two pdf's (shaded area of our Fig. 1) is also meaningless, and certainly *not* equal to PF in the general case. Consider for instance the case in which $f_d(x)$ is wholly *on the right* of $f_c(x)$ (ship load is always greater than ship strength). In this case, there is no overlap, hence Step 2 produces PF = 0, whereas in reality PF = 1 (a rather serious miscalculation).

Other counterexamples pointing out this fallacy can be constructed. Take for instance the case in which the two pdf's are *identical*. In this case, the correct PF is equal to ½, since it is equally likely for x to be above y or below y. However, since the overlap of the two pdf's is total, Step 2 erroneously produces PF = 1.

Looking at this from another perspective, this fallacy is not surprising. Mathematically, the area of overlap is equal to $\int_0^\infty \min(f_d(x), f_c(x))\, dx$. But this is a quantity that has no real meaning from a probabilistic viewpoint.

(*c*) The first counterexample above can also be used to show that the integral $\int_0^\infty f_d(x)f_c(x)dx$ (Step 3) is meaningless. This

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is so not because the integral is *not* generally equal to the area of overlap between the two pdf's, but more important, because it is not equal to PF in the general case (the integral yields PF = 0 instead of 1 in the first counterexample above).

(*d*) Given the above method is erroneous, its special cases involving Dirac functions are also wrong, for obvious reasons. Equation PF = $f_d(x_2)$ is meaningless anyway (f_d could be zero at $x = x_2$). In addition, PF is 1, and not 0, if $x_1 > x_2$.

The above mean that one may dramatically underestimate or overestimate PF if this method is used. Which is then a correct way?

To calculate PF, we first note that PF = prob($x \geq y$). This probability can be computed by integrating the joint *pdf* of x and y (equal to the product of the two individual pdf's because x and y are independent) over the sample space defining the event of failure ($x \geq y$, see Fig. 2). Then it is straightforward to see that

$$PF = \int_0^\infty f_d(x) \int_0^{(x)} f_c(y) dy\, dx \tag{1}$$

In closing, it is interesting to note that in another part of [1], Chang presents another formula for PF which is correct, and essentially identical to our equation (1). This is equation (44) of [1] (page 311), in which PF = $\int_0^\infty f_d(x) F_c(x) dx$, where

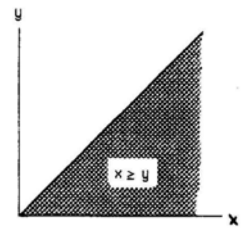

Fig. 2

F_c is the (cumulative) distribution function of y. The fact that this formula is identical to our equation (1) can be understood by the fact that $F_c(x) = \int_0^x f_c(y) dy$.

It is hoped that this technical note will resolve possible confusions in the state of knowledge in this area that may occur because of the aforementioned erroneous way of calculating PF.

Reference

1 Chang, P. Y., "A State-of-the-Art Review of the Reliability Approach and Methodology for the Design of Aerospace and Ocean Systems," Marine Technology, Vol. 27, No. 5, Sept. 1990, pp. 300–320.