Journal of Ship Research, Vol. 22, No. 3, Sept. 1978, pp. 186–192

Some New Aspects of Slamming Probability Theory

Harilaos N. Psaraftis[1]

A systematic investigation of some probabilistic aspects of slamming is presented. This investigation includes the assessment of the unconditional probability of slamming at a random instant of time; the estimation of the conditional probability of slamming at a given instant after a particular slam; and the consequent rejection of the hypothesis that slamming is a Poisson process. In addition, a procedure to approximate the distribution of slamming interarrival times[2] is presented. Finally, new slamming statistics, obtainable from the theory of this work, are presented and compared with the existing slamming criteria. The theory of this paper can be readily applied to other seakeeping events such as deck wetness, keel emergence, and propeller racing.

Introduction

SLAMMING STUDIES can be classified into two equally important categories:

(*a*) Hydrodynamic and structural analyses of the impact phenomenon itself.

(*b*) Probabilistic and statistical analyses of the occurrence of that phenomenon.

Without trying to underestimate the importance and implications of a hydrodynamic and structural analysis of slamming, this paper belongs to the second category. In this respect it will investigate the slamming problem, starting from the very simple question of "how often" slamming occurs and proceeding with more subtle issues on the mechanisms governing the process.

Szebehely [1][3] and Tick [2] assume a ship to be slamming in an interval of time $(t, t + dt)$ if, during that interval

1. the ship keel at a prescribed location (the slamming station) is just entering the sea surface,
2. the relative vertical velocity between ship and sea surface at the slamming station is greater than a critical velocity v_0, and
3. the relative angle between keel and sea surface at the slamming station and at the instant of keel entry is "small."

Today the most widely accepted probabilistic slamming model requires only the first two conditions to be true; so does the theory developed in this work.

The slamming station is taken to be in the vicinity of the forward perpendicular, usually at Station 1, *LBP*/20 aft of the forward perpendicular. The value of v_0, the so-called "threshold velocity," is usually taken to be 12 fps (3.657 m/s) for a vessel of 520 ft (158.5 m) *LBP* (Ochi [3]). For different ship lengths, Froude-law scaling is suggested. Aertssen [4] suggests a value of 18 fps (5.486 m/s) for a 520-ft ship length.

In order to be in a position to extract slamming statistics from the model just described, the following assumptions are also made:

1. The wave elevation at any point and at any instant is a stationary, narrow-band, zero-mean Gaussian random variable.
2. The ship is a linear, time-invariant system for which the input is the sea excitation and the outputs are the ship motions.

A consequence of these assumptions is that all the ship motions are also stationary, narrow-band, zero-mean Gaussian random variables. The same holds for the relative vertical motion and the relative vertical velocity between the ship and the sea surface. Also, the joint probability density function of any number of these variables (even of variables separated by a time interval) is a multivariate Gaussian one.

The theory that follows assumes also long-crested unidirectional irregular head seas. This assumption is made only to match the conditions of the full-scale slamming measurements on the SS *Wolverine State* [8] and therefore causes no loss of generality.

Slamming probabilities: unconditional and conditional

From [2, 3, 5] the a priori (unconditional) probability of the ship slamming in the interval $(t, t + dt)$, where t is random and dt small, is shown to be equal to $dP = \lambda dt$, where

$$\lambda = \frac{1}{2\pi}\sqrt{\frac{-\psi''(0)}{\psi(0)}}\, e^{-1/2[k^2/\psi(0) - v_0{}^2/\psi''(0)]} \qquad (1)^4$$

where

k = draft at slamming station

v_0 = threshold velocity

$\psi(t)$ = autocorrelation function of relative vertical motion $X_1(t)$ between sea surface and ship at slamming station

$$= \int_0^\infty \cos\omega_e t\,|R_1(\omega_e)|^2 S(\omega)d\omega$$

$\omega_e = \omega + \dfrac{\omega^2 V}{g}$ = frequency of encounter for head seas

V = ship speed

$R_1(\omega_e)$ = complex response amplitude operator of $X_1(t)$

g = acceleration of gravity

$S(\omega)$ = energy density spectrum of seaway

Primes in equation (1) denote time differentiation and $\psi(0)$ and $-\psi''(0)$ are the variances of $X_1(t)$ and $\dot{X}_1(t)$, respectively.

Slamming was assumed by Ochi [3, 6] to be a Poisson process with parameter λ. A consequence of this assumption is that the probability mass function (*PMF*) for r, the exact number of slams in an interval of T, is given by

$$P(r,T) = \begin{cases} \dfrac{(\lambda T)^r e^{-\lambda T}}{r!} & r = 0, 1, 2 \ldots \text{ and } T > 0 \\ 0 & \text{otherwise} \end{cases} \qquad (2)$$

[1] Graduate student and research assistant, (now research associate) Department of Ocean Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

[2] The expressions "slamming interarrival time," "slamming interval," and "time interval between two successive slams" are equivalent and are used interchangeably throughout this paper.

[3] Numbers in brackets designate References at end of paper.

Manuscript received at SNAME headquarters March 9, 1977; revised manuscript received November 16, 1977.

[4] Derivations of formulas for λ when all three conditions of slamming are taken into account are presented in references [2, 5].

Reprinted from JOURNAL OF SHIP RESEARCH, Vol. 22, September 1978, pp. 186-192

It was felt, however, that a ship cannot slam more frequently than its natural calm-water pitch period t_*. Therefore, Ochi modified the exponential probability density function (*PDF*) of the slamming interarrival times that would hold for a Poisson process [7]

$$f_t(t) = \begin{cases} \lambda e^{-\lambda t} & t \geq 0 \\ 0 & t < 0 \end{cases} \tag{3}$$

as follows:

$$f_t{}^*(t) = \begin{cases} \lambda e^{-\lambda(t-t_*)} & t \geq t_* \\ 0 & t < t_* \end{cases} \tag{4}$$

From a theoretical point of view, however, using (1) in both (2) and (4) would create a complication. This can be understood from the fact that while (4) implicitly forbids more than two slams to occur in a time interval shorter than t_*—in fact, regarding such an event impossible—equation (2), on the other hand, explicitly allows any number of slams in any time interval, with a finite probability, even if this interval is shorter than t_*.

It should be mentioned at this point that the value of λ used by Ochi to test (4) was not the one suggested by equation (1), but simply the one that was derived from the experimental data itself, namely, the number of slams divided by the time interval during which the observations took place. These observations were made aboard the SS *Wolverine State* [8].

Concerning now the validity of (2) for the slamming process, it turns out that there is a fundamental property that a Poisson process has and it is not obvious at first glance whether this property belongs also to slamming or not. This is the "no memory" property, namely, the mutual independence of arrivals. To check therefore the validity of (2) for slamming, we have to check whether slamming has memory or not.

Our approach to check this will be to test if the conditional probability of a slam in $(t, t + dt)$, given a slam at time zero, is still equal to λdt. If it is, this would mean that the occurrence of a slam at $T = 0$ is irrelevant for the probability of a slam thereafter. Since this is synonymous with lack of memory, we conclude that slamming will be a Poisson process if and only if the foregoing conditional probability is equal to λdt.

Let us name this conditional probability $\phi(t)dt$.

To evaluate $\phi(t)$, we further define event $A(t)$ as the occurrence of a slam in $(t, t + dt)$. Then

$$\phi(t)dt = \text{prob}\{A(t)|A(0)\} = \frac{\text{prob}\{A(0)A(t)\}}{\text{prob}\{A(0)\}} \tag{5}$$

Earlier we saw that prob$\{A(0)\}$ = prob$\{A(t)\} = \lambda dt$. Expressing the conditions for $A(t)$ mathematically, for some instant of time τ in $(t, t + dt)$, we have

$$A(t)\text{:}\ \{X_1(\tau) = -k \text{ and } X_2(\tau) \geq v_0\}$$

where $X_1(t)$ is the relative vertical motion between the sea surface and the ship (positive when bow is down) and $X_2(t) \equiv \dot{X}_1(t)$.

Examining what happens at t and at $t + dt$, we note that

$$X_1(t) \leq -k \text{ (bow has not touched the surface)}$$

and that

$$X_1(t + dt) \geq -k \text{ (bow is already in the water)}$$

If dt is small

$$X_1(t + dt) \simeq X_1(t) + X_2(t)dt$$

and

$$X_2(t) \geq v_0$$

So

$$A(t)\text{:}\ \{-k - X_2(t)dt \leq X_1(t) \leq -k \text{ and } X_2(t) \geq v_0\}$$

Then (5) yields

$$\phi(t)dt = \frac{1}{\lambda dt}\,\text{prob}\{-k - X_2(T)dt \leq X_1(T) \leq -k \text{ and } X_2(T) \geq v_0 \text{ for } T = 0 \text{ and } t\} \tag{6}$$

The four random variables in the brackets, $\xi_1 \equiv X_1(0)$, $\xi_2 \equiv X_1(t)$, $\xi_3 \equiv X_2(0)$, and $\xi_4 \equiv X_2(t)$, have a four-variate joint Gaussian distribution, $F_4(\xi_1, \xi_2, \xi_3, \xi_4)$.

So, equation (6) yields

Nomenclature

$A(t)$ = event: a slam occurs in time interval $(t, t + dt)$
$B(t)$ = event: no slam occurs in time interval (dt, t)
C = covariance matrix for multivariate Gaussian *PDF*
C^{-1} = inverse of C
D = determinant of C
$E(x)$ = expected value of random variable x
$f(t)$ = *PDF* for time interval between two successive slams
$f_t(t)$ = exponential *PDF* for time interval between two successive slams
$f_t{}^*(t)$ = Ochi's truncated *PDF* for time interval between two successive slams
$F_N(X_1, X_2, \ldots X_N)$ = *N*-variate joint Gaussian *PDF*
g = acceleration of gravity
$\overline{H}^{1/3}$ = significant wave height
k = draft at slamming station
LBP = length between perpendiculars
prob$\{\ldots\}$ = probability
$p_N(T)$ = probability of *N* slams separated from each other by an interval $\leq T$
PDF = probability density function (for continuous variables)
PMF = probability mass function (for discrete variables)
$P(r,T)$ = Poisson *PMF* for r, the exact number of arrivals in an interval T
$R_1(\omega_e)$ = complex response amplitude operator of $X_1(t)$
$S(\omega)$ = energy density spectrum of seaway
t, T = time instant or interval
t_* = natural calm-water pitch period of ship
t_0 = most probable slamming interval
$\overline{T}$ = mean period of response spectrum of $X_1(t)$
V = ship speed
v_0 = critical velocity for slamming
$X_1(t)$ = relative vertical motion between sea and ship at slamming station
$X_2(t)$ = $dX_1(t)/dt$
λ = a priori slamming probability per unit time, slamming frequency of occurrence
σ_{ij} = (i,j)th element of C
σ^{ij} = (i,j)th element of C^{-1}
σ_x = standard deviation of random variable x
τ = time instant
$\phi(t)$ = per unit time probability of a slam at $T \simeq t$ given a slam at $T \simeq 0$
$\psi(t)$ = autocorrelation function of $X_1(t)$
ω = circular wave frequency
ω_e = frequency of encounter
ω_p = frequency of spectral peak

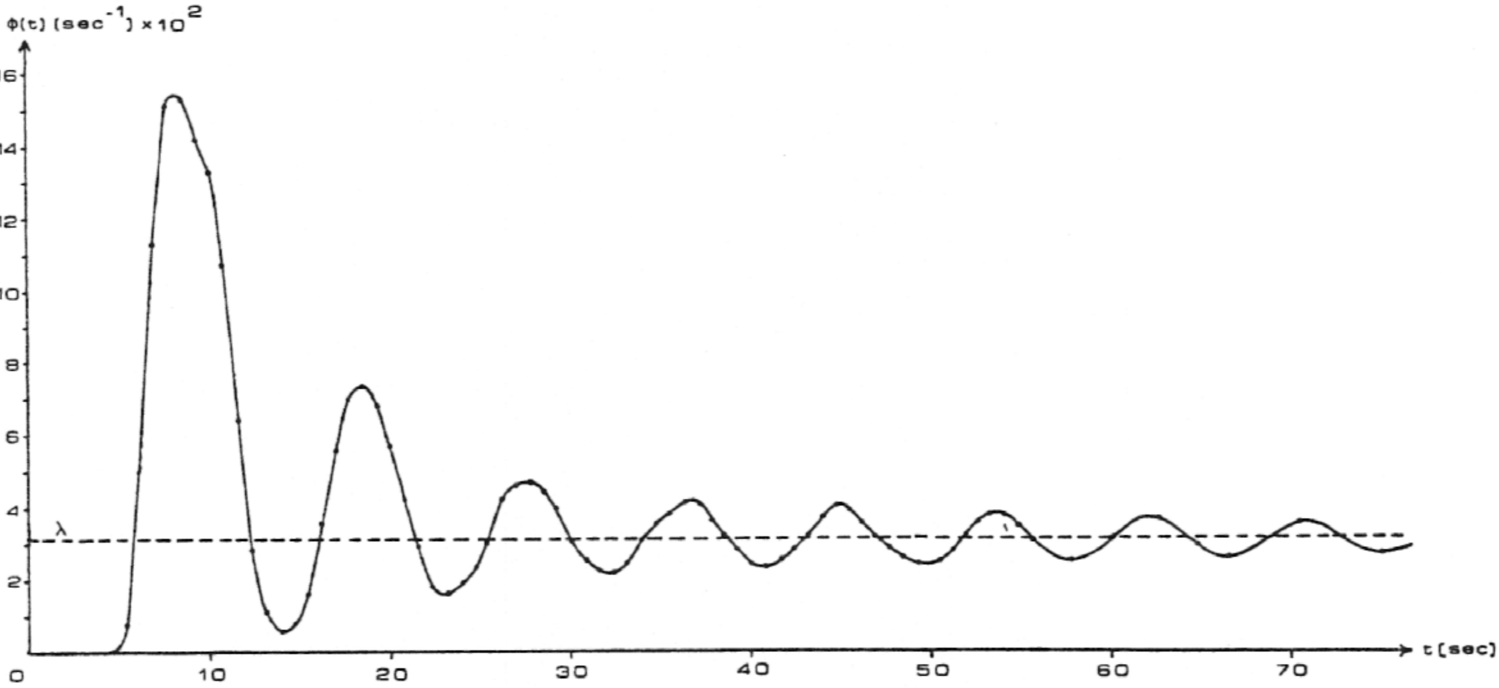

Fig. 1 Plot of $\phi(t)$ for the SS *Wolverine State* (Voyage 288, Interval 58); λ was calculated from equation (1) to be 0.032 s^{-1}

$$\phi(t)dt$$

$$= \frac{1}{\lambda dt}\int_{v_0}^{\infty}\int_{v_0}^{\infty}\int_{-k-\xi_4 dt}^{-k}\int_{-k-\xi_3 dt}^{-k} F_4(\xi_1,\xi_2,\xi_3,\xi_4)d\xi_1 d\xi_2 d\xi_3 d\xi_4$$

or, finally

$$\phi(t) = \frac{1}{\lambda}\int_{v_0}^{\infty}\int_{v_0}^{\infty} \xi_3\xi_4 F_4(-k,-k,\xi_3,\xi_4)d\xi_3 d\xi_4 \qquad (7)$$

F_4 is given by (see [9])

$$F_4(\xi_1,\xi_2,\xi_3,\xi_4) \equiv \frac{1}{(2\pi)^2 D^{1/2}} e^{-1/2\sum_{i=1}^{4}\sum_{j=1}^{4}\xi_i\xi_j\sigma^{ij}}$$

where σ^{ij} are the elements of C^{-1}, the inverse of C, the covariance matrix of ξ_1,ξ_2,ξ_3,ξ_4, and D is the determinant of C.

Application of linear systems analysis to ship responses [5] yields that the elements σ_{ij} of C are given by

$$\sigma_{11} = \sigma_{22} = \psi(0) \qquad \sigma_{14} = \sigma_{41} = \psi'(t)$$

$$\sigma_{13} = \sigma_{31} = \sigma_{24} = \sigma_{42} = 0$$

$$\sigma_{12} = \sigma_{21} = \psi(t) \qquad \sigma_{23} = \sigma_{32} = -\psi'(t)$$

$$\sigma_{33} = \sigma_{44} = -\psi''(0) \qquad \sigma_{34} = \sigma_{43} = -\psi''(t)$$

It can be seen that the method used to derive the expression for $\phi(t)$ (7) is similar to the one used by Longuet-Higgins [10] for the derivation of the conditional probability of a stationary random Gaussian variable having a zero crossing in $(t, t + dt)$, given that it had one in $(0, dt)$. In fact, the zero crossing problem is a subproblem of the slamming problem, which is a $-k$ crossing problem for $X_1(t)$ under the further restriction that each $-k$ crossing has to have $\dot{X}_1(t) \geq v_0$.

Figure 1 represents a picture of $\phi(t)$ derived by numerical evaluation of (7) for the *Wolverine State*, Voyage 288, Interval 58 [8] (see the Appendix). One can clearly note that $\phi(t)$ is far from being a constant equal to λ, as it ought to be if slamming were a Poisson process. It can be seen, therefore, that slamming cannot be considered as a Poisson process and consequently (2) cannot hold.

Since slamming is not a Poisson process, (4) cannot hold either, because (4) was derived by modification of the interarrival time *PDF* of a process which was assumed to be Poisson. The fact that (4) cannot hold was to be expected because a ship in irregular seas may do anything but pitch regularly at its natural calm-water pitch period. Incidentally, experiments aboard the *Wolverine State* [8] showed five out of 117 slamming intervals being shorter than the natural calm-water pitch period of the ship.

From the shape of $\phi(t)$ it can be seen that $\phi(t)$ tends to λ for large values of t. This property was also to be expected because no correlation exists between seaway events separated by a long time interval. So the fact that a slam has occurred at $T \simeq 0$ may be very important for the likelihood of another slam 10 s later, but it is completely irrelevant for the probability of a slam after 10 min. So the conditional probability should converge to the a priori (unconditional) probability.

An analytical verification of this property is the following:

As $t \to \infty$, $\psi(t)$ and its derivatives tend to zero. So the only nonzero elements of C are $\sigma_{11} = \sigma_{22} = \psi(0)$ and $\sigma_{33} = \sigma_{44} = -\psi''(0)$. With such a decoupling, it is easy to see that the double integral in (7) is λ^2, where λ is given by (1). So

$$\phi(t) \to \lambda \text{ as } t \to \infty$$

The exact behavior of $\phi(t)$ as $t \to 0$ is more difficult to determine because at $t = 0$, C becomes singular. The reader is referred to [10, 11] for methods to overcome such difficulties. In this work we will contend ourselves with the results from the calculations we made, which show $\phi(t)$ to be very small at small t's. This also should have been anticipated, for it seems very unlikely (but not necessarily impossible) to have another slam very close to the slam at $T \simeq 0$.

For intermediate values of t, $\phi(t)$ is seen to oscillate. This means that the ship is more likely to slam again at integer multiples of some characteristic period t_0 namely, t_0, $2t_0$, $3t_0$, etc., and less likely at $t_0/2$, $3t_0/2$, $5t_0/2$. This "periodicity" tends to die off as t increases, as the correlation of seaway events separated by a longer time interval gets weaker.

The time interval between two successive slams

Let $f(t)$ be the *PDF* for the slamming interarrival time. Earlier we saw that $f(t)$ is neither an exponential (Poisson) *PDF* (3) nor an Ochi-truncated *PDF* (4). Here we will see how we can estimate $f(t)$ from the information we have so far.

The problem of finding $f(t)$ of a random function, whether i

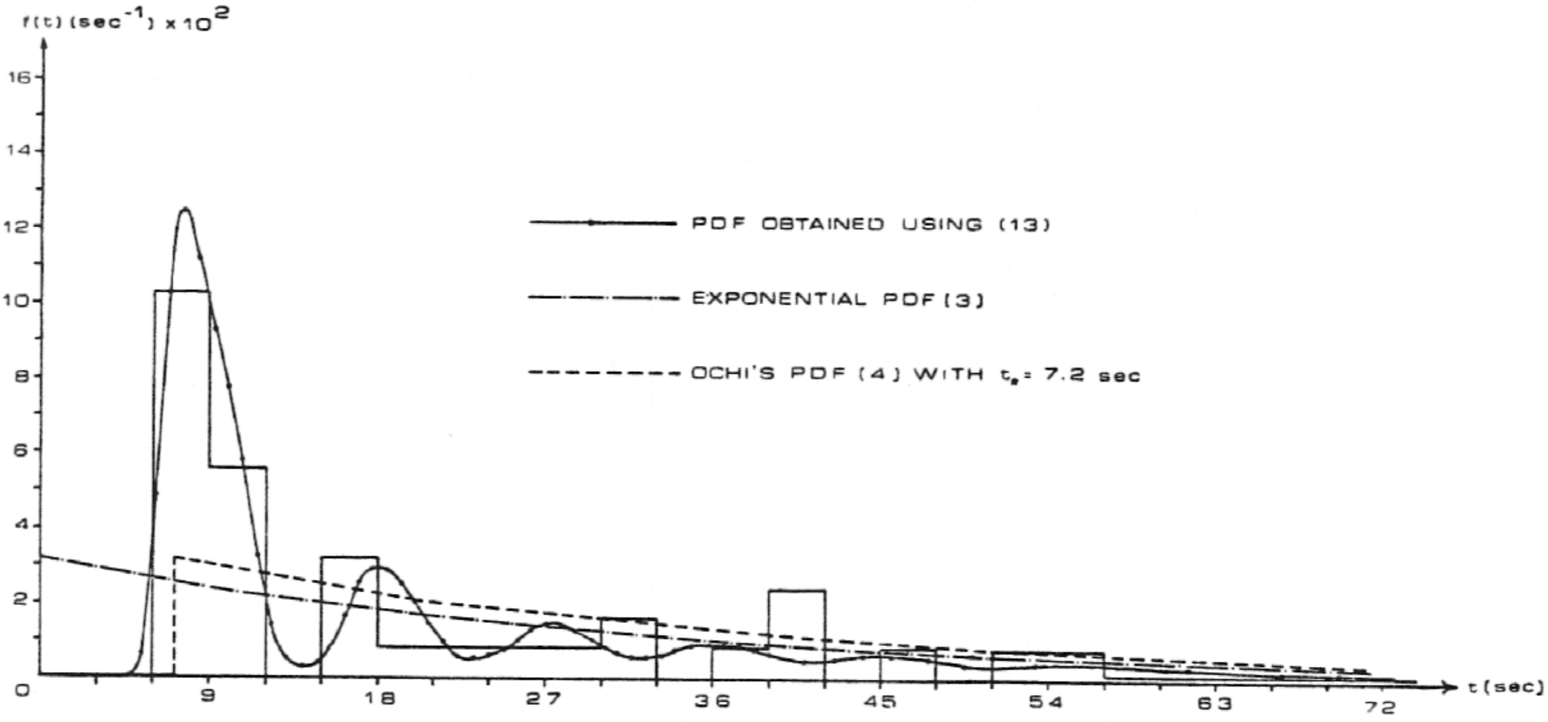

Fig. 2 Comparison of the histogram of the distribution of slamming intervals for the SS *Wolverine State* (Voyage 288, Interval 58) with the *PDF* obtained using equation (13), the exponential *PDF* (3), and Ochi's *PDF* (4) with $t_* = 7.2$ s. All *PDF*'s use $\lambda = 0.032\ s^{-1}$

means interval between successive slams or interval between successive zeros, up-crossings or down-crossings, maxima or minima, has many physical applications which can be found in studies of electrical circuits, microseisms, irregularities in the ionosphere, sea waves, etc. However, a survey of the relevant literature [10–14] reveals that only approximate solutions have been found even in the most simple case of Gaussian variables.

A first-order approximation of $f(t)$ is simply $\phi(t)$. This approximation is valid for small values of t only [14]. In fact, if t is small, a slam in $(t, t + dt)$ is most likely to be the first one after the one that occurred in $(0, dt)$. Nevertheless, it can be seen that this approximation violates the fundamental probability axiom, because $\int_0^\infty \phi(t)dt$ is not equal to unity.

In this paper we give an improved, second-order approximation for $f(t)$.

Let us, in addition to the already defined event $A(t)$, define event $B(t)$ to signify the absence of a slam between $T = dt$ and $T = t$. Then, from [7], we have

$$\text{prob}\{B(t)A(t)|A(0)\} = \text{prob}\{B(t)|A(0)\}\text{prob}\{A(t)|A(0)B(t)\} \quad (8)$$

If the slam that occurred in $(0, dt)$ is the zeroth slam, then

$$\begin{aligned}\text{prob}\{B(t)A(t)|A(0)\} &= \text{prob}\{\text{time interval between zeroth slam and first slam is between } t \text{ and } t + dt\}\\ &= f(t)dt \qquad (9)\end{aligned}$$

Also,

$$\begin{aligned}\text{prob}\{B(t)|A(0)\} &= \text{prob}\{\text{first slam will occur later than } t \text{ after zeroth slam}\}\\ &= \int_t^\infty f(x)dx \qquad (10)\end{aligned}$$

The probability $\text{prob}\{A(t)|A(0)B(t)\}$ [conditional probability of a slam in $(t, t + dt)$ given a slam in $(0, dt)$ and no slam in (dt, t)] cannot be expressed in closed form. In this paper we will approximate this probability with $\phi(t)dt$, that is

$$\text{prob}\{A(t)|A(0)B(t)\} \simeq \text{prob}\{A(t)|A(0)\} = \phi(t)dt \quad (11)$$

Putting (9), (10), and (11) into (8), we end up with an integral equation for $f(t)$:

$$f(t) = \phi(t) \int_t^\infty f(x)dx \quad (12)$$

The solution of (12) which obeys the condition

$$\int_0^\infty f(t)dt = 1$$

is

$$f(t) = \begin{cases} \phi(t)e^{-\int_0^t \phi(x)dx} & t \geq 0 \\ 0 & t < 0 \end{cases} \quad (13)$$

We can see that for small values of t, (13) reduces to the earlier established first-order approximation $f(t) \simeq \phi(t)$.

Figure 2 is a plot of $f(t)$ derived for the *Wolverine State*, Voyage 288, Interval 58, by numerical application of (13). In the same figure one can see the histogram of the slamming intervals obtained from the observations aboard the ship [8] and also the exponential and Ochi-truncated *PDF*'s for a process with parameter λ derived using (1). Ochi's *PDF* was conditioned at 7.2 s, the natural calm-water pitch period of the ship. One can see the good agreement of (13) with the histogram.

Also, one can note the similarity of $f(t)$ (Fig. 2) with $\phi(t)$ (Fig. 1) when t is small. This similarity disappears at large t's.

At large t's, $\phi(t) = \lambda$ for $t > t_1$, so that $f(t)$ behaves like $Ae^{-\lambda t}$ where

$$A = \lambda e^{\lambda t_1 - \int_0^{t_1} \phi dx} = \text{constant}$$

It is interesting also to note that (13) reduces to (3) or (4) depending on the appropriate choice for $\phi(t)$. For a Poisson process, substituting $\phi(t) = \lambda$ into (13) yields immediately (3). For an Ochi-truncated process, $\phi(t) = \lambda$ for $t \geq t_*$ and zero otherwise, and we end up with (4).

Approximations of higher order than (13) are given by Rice [14], McFadden [11], and Longuet-Higgins [10]. In references [10,14], $f(t)$ is given as an infinite series and in [11] as a convolution integral. We note that the computational effort associated with these higher-order approximations can be substantial.

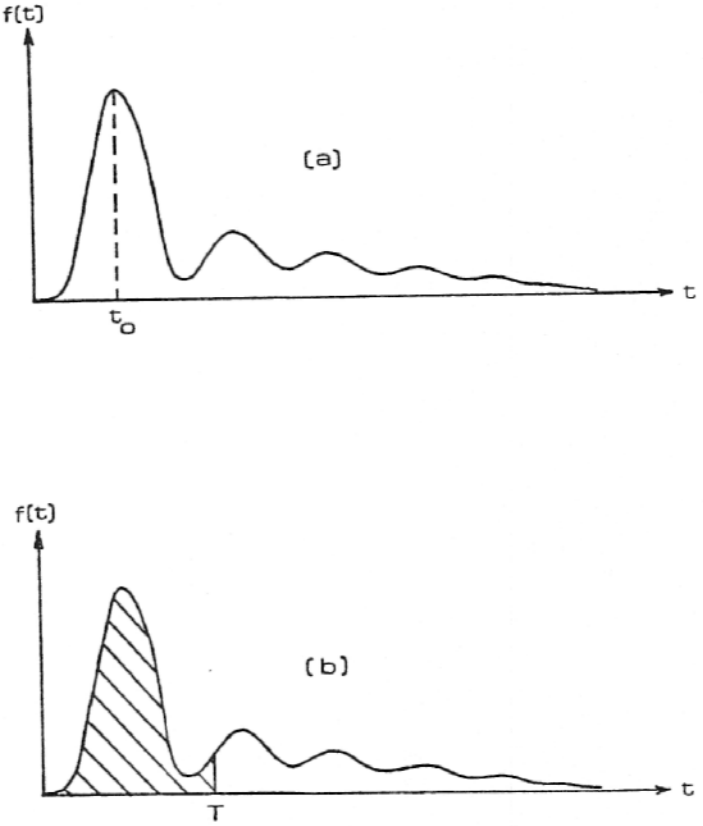

Fig. 3 Two slamming statistics: (*a*) most probable slamming interval t_0, and [shaded area in (*b*)] probability of a slamming interval shorter than T

Application of properties of the Laplace transform of $f(t)$ [7] can yield the expected value $E(t)$ and variance σ_t^2 of the slamming interarrival times. If $f(t)$ obeys (13), then (see also [5])

$$E(t) = \int_0^\infty e^{-F(t)}dt \tag{14}$$

$$\sigma_t{}^2 = 2\int_0^\infty te^{-F(t)}dt - [E(t)]^2 \tag{15}$$

where, by definition

$$F(t) \equiv \int_0^t \phi(x)dx \tag{16}$$

Selecting a meaningful slamming statistic

The traditional statistic used for slamming predictions is the expected value of the number r of slams in a given time interval T.

For this statistic to be useful, one would also need some information about σ_r, since any prediction based only on averages could be misleading. Now σ_r would be readily obtainable if slamming were considered a Poisson process. Then we would have $E(r) = \lambda T$ and $\sigma_r = \sqrt{\lambda T}$. But slamming is not a Poisson process and therefore we cannot use the foregoing formula for σ_r. It turns out that it is very difficult to extract σ_r for a non-Poisson process. $E(r)$ can still be calculated [5] but having only $E(r)$ as a statistic would not be of help.

Fortunately, it is not necessary to proceed to fulfill this goal, since it is difficult to relate $E(r)$ and σ_r to the captain's decision to reduce speed or change heading because of slamming. There are other statistics more closely connected to that decision and which are readily available from the theory of this work.

For example, St. Denis [15] suggested as a criterion for slamming the likelihood of the occurrence of a closely spaced succession of slams. This criterion is consistent with the captain's reactions when the ship experiences slamming. A captain will reduce speed if the ship encounters, say, three consecutive slams at short intervals. If such a succession happens, he will reduce speed or possibly change heading right after it, rather than maintain speed in order to count the exact number of slams in a prescribed time interval and then react.

A first statistic connected to the slamming interarrival time is the most probable slamming interval t_0. It is the time interval corresponding to the first (and highest) peak of $f(t)$, or, since $f(t) \simeq \phi(t)$ when t is small, to the first (and highest) peak of $\phi(t)$ [Fig. 3(*a*)].

Thus, a ship displaying an $f(t)$ curve with a small value of t_0 will be more susceptible to rapid slam successions than a ship for which t_0 has a large value.

The results of the calculations for the *Wolverine State* (Appendix of [5]) showed that t_0 was in the close vicinity of $\overline{T}$, the mean period of the response spectrum of the relative motion at the slamming station. This result may establish a convenient rule of thumb for t_0 since $\overline{T}$ can be obtained in closed form:

$$\overline{T} = 2\pi\sqrt{\frac{-\psi(0)}{\psi''(0)}} \tag{17}$$

Another useful statistic is the probability that the ship will encounter two successive slams in a time interval shorter than a known interval T. This is equal to the shaded area in Fig. 3(*b*), or

$$p_2(T) = \int_0^T f(t)dt \tag{18}$$

It can be seen from Fig. 2 that both the exponential (Poisson) and the Ochi-truncated *PDF*'s tend to underestimate this probability because at small t's they are not as peaked as the $f(t)$ derived in this work and the histogram ordinates.

If we assume that slamming interarrival times are mutually independent, then we can extend equation (18) to more slams. For example, the probability that a sequence of N slams separated from one another by an interval smaller than T is

$$p_N(T) = [p_2(T)]^{N-1} \qquad N = 2, 3, 4 \ldots \tag{19}$$

This assumption implies that slamming has memory only up to the previous slam. This hypothesis reminds us of a Markov process where the memory of the process extends only one step into the past [7]. A similar assumption was made by McFadden [11].

If this assumption is dropped, $p_N(T)$ can be obtained in a more complicated way using higher-order interarrival time *PDF*'s.

This last criterion (19) has the closest correspondence to the suggestion of St. Denis [15] than any criterion encountered so far.

What proved to be a surprising property of the theory presented in the foregoing was the fact that the location of the maxima and minima of $\phi(t)$ turned out to be independent of the value of k and the assumed value of the threshold velocity v_0 [5]. The results that substantiate this statement are shown in Fig. 4.

This result enhances the value of t_0 as a slamming statistic. Estimates of t_0 can be obtained using [17] (since $t_0 \simeq \overline{T}$) without the uncertainty associated with v_0. Of course, for more information about the process, k and v_0 are highly important and should not be ignored.

A complete sensitivity analysis of all the parameters affecting slamming is presented in [5].

Conclusions

This work has focused on three interrelated issues:

1. The investigation of whether or not slamming is a Poisson process: In this respect, the conditional probability of slamming per unit time was considered and the Poisson hypothesis was consequently disproved.

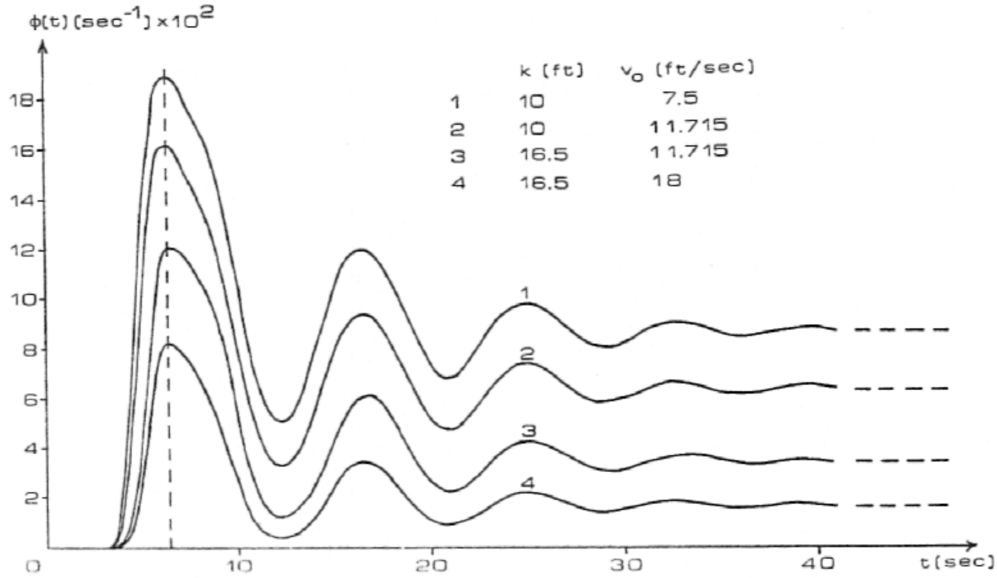

Fig. 4 Insensitivity of location of maxima and minima of $\phi(t)$ to changes in the assumed values of k and v_0. The plots were done for the SS *Wolverine State* with a two-parameter spectrum of $\bar{H}^{1/3}$ = 28 ft (8.53 m) and ω_p = 0.725 rad/s assumed, and a ship speed of 2 knots

2. The extraction of the distribution of the slamming inter-arrival times: An approximate solution was presented in this paper and was shown to compare favorably with experimental data.

3. The recommendation of more meaningful slamming statistics: It was felt that these statistics and the underlying theory behind them can be used for a better assessment of the performance of a ship in a seaway.

The procedures for the extraction of $f(t)$ vary in their degree of accuracy and computational effort. It was seen that $f(t)$ can provide information leading to new and more meaningful slamming statistics. In particular, the most probable slamming interval t_0 is a valuable statistic since it is easy to compute (17) and is independent of v_0 and k.

The theory presented in this paper can be similarly applied to every other seaway event that can be characterized with the exceedance or crossing of prescribed levels of random seakeeping variables. Deck wetness, keel emergence, and propeller racing are three events that fit into this category.

Finally, the approach of this work can be readily extended to non-head unidirectional or even short-crested multidirectional seas.

Acknowledgments

Sincere thanks are due to Professor C. Chryssostomidis for his continuous help, to Professor T. A. Loukakis for his assistance during the formulation of the problem, and to Dr. M. K. Ochi for his comments. Thanks are also due to the Department of Ocean Engineering at M.I.T. for its financial assistance and to the National Science Foundation for the funds that made the computations of this paper possible.

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Appendix

Data used for experimental verification

The data of Voyage 288, Interval 58, of the SS *Wolverine State* [8] were chosen to test the theory of this work. This interval was chosen because at that time the ship was slamming the most.

It was decided [Appendix D of [5]] to represent the seaway spectrum as a superposition of the following two spectra:

(*a*) A Bretschneider spectrum of $\bar{H}^{1/3}$ = 21.64 ft (6.59 m) and ω_p = 0.725 rad/s. This spectrum accounts for the energy in the "sea" component of the seaway.

(b) A Bretschneider spectrum of $\overline{H}^{1/3} = 30.44$ ft (9.27 m) and $\omega_p = 0.567$ rad/s truncated so as to be valid only between 0.378 and 0.685 rad/s. This spectrum accounts for the energy in the "swell" component of the seaway.

The slamming station was taken to be at the low-pressure transducer LP21, 228 ft (69.49 m) forward of amidships. This transducer was used in [8] to detect slamming intervals.

The critical velocity v_0 was assumed equal to 11.715 fps (3.57 m/s), which is what Ochi's Froude scaling hypothesis yields for a ship of 496 ft (151.18 m) *LBP*.

Head seas and a forward speed of 3.38 fps (1.03 m/s), which is what was recorded in the data log (2 knots), were assumed.

The M.I.T. five-degree-of-freedom seakeeping program was used to provide the ship responses for heave and pitch.

The values of λ, $\phi(t)$, and $f(t)$ were computed from equations (1), (7), and (13) respectively. All spectral integrations involving $\psi(t)$ were truncated at seven times the frequency of the "sea" spectrum peak. This truncation scheme was tested in reference [5], Appendix C, and was found satisfactory.

Forty-three slams were detected in a duration of 1210 s. The corresponding 42 slamming intervals are (page 9 of [8]) (seconds):

11.4	8.0	8.5
8.0	25.0	79.5
39.1	66.5	51.0
28.9	21.0	9.0
8.2	62.0	8.0
41.2	45.5	8.0
41.4	8.5	10.0
9.5	8.0	7.0
15.5	8.5	55.0
30.5	127.5	10.0
20.0	15.0	14.0
16.0	90.0	7.0
36.0	7.5	9.0
7.5	31.5	10.0

The histogram of the distribution of these intervals appears in Fig. 2.

The average number of slams resulting is either $43/1210 = 0.035$ slams/second or $43/1134.4 = 0.038$ slams/second, depending on what is the total time interval considered ($75.6 = 1210 - 1134.4$ was the interval from the time the experiments started up to the first slam). Application of (1) yielded $\lambda = 0.032$ slams/second.